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Progressive Data-Driven modification for Spalart-Allmaras Turbulence Modeling

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Abstract: Artificial intelligence (AI) techniques have been widely applied in turbulence modeling, but generalization ability and physical interpretability remain core challenges for their application in complex engineering turbulence simulations. In recent years, the concept of progressive modeling has been proposed, where new corrections to the model do not interfere with the calibration of the original model, significantly improving its generalization capability. We extend this concept to the modification of the Spalart-Allmaras (SA) model, progressively embedding physical knowledge into the SA model from two aspects: near-wall scaling, and adverse pressure gradient (APG) effects. During the modeling process, data from large eddy simulation (LES) of zero pressure gradient flat plate, and APG flat plate are used. Physical constraints such as asymptotic laws and logarithmic laws were incorporated, and symbolic regression methods were applied to derive the algebraic expressions of the correction terms, achieving a deep integration of physics and data. Firstly, we conducted near-wall eddy viscosity scaling analysis for the SA model and established a new near-wall damping function with the correct scaling. Then, we addressed the adverse pressure gradient effect by modify destruction and production terms of SA model. The proposed method ensures that the new corrections do not degrade the accuracy of the existing corrections. While maintaining the accuracy of standard wall turbulence predictions, the method significantly improves the prediction accuracy of SA model adverse pressure gradient separation flows.

Keywords: Turbulence modeling, Data-driven, Progressive modeling, Symbolic regression

1. Introduction

Accurate turbulence modeling remains a cornerstone in computational fluid dynamics (CFD), particularly for engineering applications involving complex flow phenomena such as adverse pressure gradients (APG) and flow separation. However, traditional Reynolds-averaged Navier–Stokes (RANS) models often result in significant limitations, especially in non-equilibrium flows where the underlying assumptions fail to capture critical physical mechanisms.

In recent years, the advent of artificial intelligence (AI) techniques has opened new avenues for turbulence model enhancement. Data-driven approaches offer promising means to recalibrate classical models using high-fidelity data from direct numerical simulation (DNS) and large eddy simulation (LES)[1]. Despite their potential, many of these methods suffer from poor generalization capability. This inherent limitation significantly restricts their utility in engineering applications where flow conditions can vary widely[2].

A particularly promising strategy to overcome these challenges is progressive modeling, where incremental corrections are introduced to the baseline SA model without compromising its original calibration[3,4]. This approach allows the model to retain its established accuracy while significantly improving predictions under other conditions.

Therefore, this study progressively leverages high-fidelity data from ZPG and APG flat-plate TBLs to systematically derive correction terms using symbolic regression. The proposed modifications are rigorously validated across multiple test cases, demonstrating enhanced accuracy in predicting APG-induced separation while maintaining robust performance for canonical ZPG flows. By integrating physical knowledge with data-driven methodologies, this work provides a framework for turbulence model development that effectively balances accuracy, generalization, and interpretability.

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2. Method

This section describes the baseline Spalart–Allmaras (SA) model and details the progressive modeling strategy employed to incorporate both near-wall scaling and APG corrections. Each step leverages high-fidelity LES data and employs symbolic regression to derive explicit correction functions. By isolating corrections in different regions of the boundary layer, the proposed approach ensures that each newly introduced term does not compromise previously established calibrations.

2.1 Spalart–Allmaras turbulence model

The one-equation Spalart–Allmaras model[5] solves a transport equation for a working variable $\tilde{\nu}$, shown schematically in Eq. (1):

$$\frac{\partial \tilde{\nu}}{\partial t}+u_j\frac{\partial \tilde{\nu}}{\partial x_j}=C_{b1}\hat{S}\tilde{\nu}-C_{w1}f_w\left(\frac{\tilde{\nu}}{d}\right)^2+\frac{1}{\sigma}\left[\frac{\partial}{\partial x_j}\left((\nu+\tilde{\nu})\frac{\partial \tilde{\nu}}{\partial x_j}\right)+C_{b2}\frac{\partial \tilde{\nu}}{\partial x_j}\frac{\partial \tilde{\nu}}{\partial x_j}\right] \tag{1}$$

where ν is the molecular kinematic viscosity, d is the distance to the nearest wall, and the basic parameters are $\sigma=\frac{2}{3}$, $C_{b1}=0.1355$, $C_{b2}=0.622$. $\hat{S}$ is a modified vorticity magnitude:

$$\hat{S}=\Omega+\frac{\tilde{\nu}}{\kappa^2 d^2}f_{v2},\ f_{v2}=1-\frac{\chi}{1+\chi f_{v1}} \tag{2}$$

where $\chi=\frac{\tilde{\nu}}{\nu}$, κ=0.41, and Ω is the vorticity magnitude defined by

$$\Omega=\sqrt{2\Omega_{ij}\Omega_{ij}},\ \Omega_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}-\frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

The eddy viscosity ν_t is computed from Eq. (4):

$$\nu_t=\tilde{\nu}f_{v1},\ f_{v1}=\frac{\chi^3}{\chi^3+C_{v1}^3},\ \chi=\frac{\tilde{\nu}}{\nu} \tag{4}$$

where $C_{v1}=7.1$. In addition, f_w function is

$$f_w=g\left(\frac{1+C_{w3}^6}{g^6+C_{w3}^6}\right)^{\frac{1}{6}},\ g=r+C_{w2}(r^6-r),\ r=\frac{\tilde{\nu}}{\hat{S}\kappa^2 d^2} \tag{5}$$

where the initial parameters in the above equations are $C_{w1}=\frac{C_{b1}}{\kappa^2}+\frac{1+C_{b2}}{\sigma}$, $C_{w2}=0.3$, $C_{w3}=2$.

The functions f_{v1} and f_w govern the behavior of the model in different flow regions. These parameters are historically calibrated for zero-pressure-gradient (ZPG) boundary layers and often require modifications to improve predictions in non-equilibrium flows—particularly those involving strong APG or separation.

2.2 Model Framework

A progressive modeling strategy is adopted to incorporate corrections into the SA model sequentially, ensuring that each step retains the successful calibration of previous steps. Figure 1 provides an overview of this framework and Eq. (6) shows the corrected terms of the transport equation. These three steps are summarized as follows:

1. **Near-wall scaling**: The function f_{v1} is refined using LES data of a ZPG turbulent boundary layer (TBL) in the viscous sublayer and buffer layer (y^+<30). This step corrects the near-wall asymptotic behavior of eddy viscosity.

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2. **Inner layer APG**: Once the near-wall scaling is established, the destruction term f_w is modified to account for APG effects using LES data from APG TBLs in the inner layer (y/δ<0.2). This modification was partially reported in our previous study[6] and focuses on improving flow predictions where strong APG can lead to flow separation.
3. **Outside the inner layer APG**: The adverse pressure gradient not only affects the inner layer but also has a significant impact on the outer layer. It increases the outer peak of the Reynolds shear stress, enhances normal convection, and excites more large-scale turbulence structures in the outer region. Therefore, to address regions beyond the inner layer (y/δ>0.2), the production term γ is further calibrated using a data-driven approach. This ensures consistency in the outer layer, preventing the corrected model from degrading in regions that are not near the wall or subject to the strongest APG influences.

Each of these steps uses **symbolic regression**[7] to identify functional forms for the correction terms. By embedding explicit physical constraints (e.g., correct near-wall asymptotic scaling, proper log-layer behavior) into the regression, we maintain interpretability and promote robust generalization across different flow regimes.

$$\frac{\partial \tilde{\nu}}{\partial t}+u_j\frac{\partial \tilde{\nu}}{\partial x_j}=\gamma C_{b1}\hat{S}\tilde{\nu}-C_{w1}f_w\left(\frac{\tilde{\nu}}{d}\right)^2+\frac{1}{\sigma}\left[\frac{\partial}{\partial x_j}\left((\nu+\tilde{\nu})\frac{\partial \tilde{\nu}}{\partial x_j}\right)+C_{b2}\frac{\partial \tilde{\nu}}{\partial x_j}\frac{\partial \tilde{\nu}}{\partial x_j}\right] \tag{6}$$

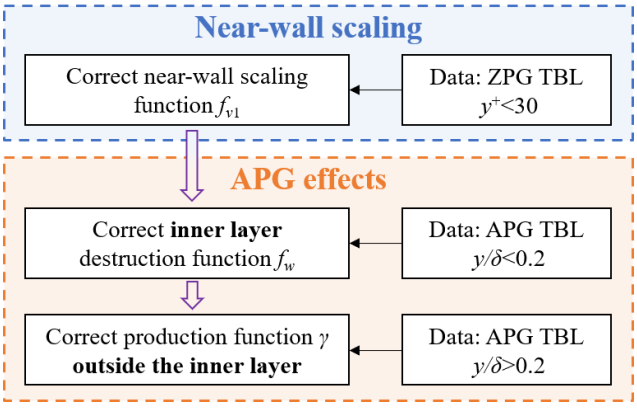

Figure 1 Framework of progressive turbulence modeling

2.3 Near-wall scaling

The near-wall behavior of eddy viscosity is crucial for accurately capturing boundary layer development. In the original SA model, the damping function f_{v1} can exhibit incorrect asymptotic trends in the viscous sublayer. To address this, we collect LES data of a ZPG TBL[8] focusing on the region y^+<30. Using symbolic regression, we then derive a modified expression for f_{v1}:

$$f_{v1}=\frac{\chi^2}{\chi^2+C_{v1,\text{Present}}^2},\quad C_{v1,\text{Present}}=8.1+0.5\tanh\left(\frac{y^+-9.2}{2.0}\right) \tag{7}$$

2.4 Adverse pressure gradient effects

Once the near-wall scaling is corrected, we address the model's deficiencies under adverse pressure gradients. We use LES data of flat-plate APG TBLs[9] to identify the destruction term f_w for inner layer shown in Eq. (8). Then, we introduce a correction γ to the production term, shown in Eq. (10). LES data from the outer region of APG TBLs[9] are employed to derive a data-driven expression.

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$$f_w = \begin{cases} g\left(\dfrac{1+C_{w3}^6}{g^6+C_{w3}^6}\right)^{\frac{1}{6}}, & 0 \le r_1 \le 1 \\ \exp\left[\left(\ln r_1 + 0.124\right)\tanh\left(\left(\dfrac{r_2}{1.25 r_1}\right)^{18}\right)\right], & 1 < r_1 \le 2 \end{cases} \tag{8}$$

$$r_1 = \frac{\tilde{\nu}}{\hat{S}\kappa^2 d^2},\ r_2 = \frac{|\nabla \tilde{\nu}|}{\hat{S}\kappa^2 d} \tag{9}$$

$$\gamma = \ln\left[\left(2r_1\right)^3 + \sqrt{r_1} + 0.043\right]\tanh\left[0.9\left(\eta - 1\right)\right] f_r \tag{10}$$

$$\eta = \left|\frac{\tau}{\tau_w}\right| = \frac{2\left(\nu + \nu_t\right)\sqrt{S_{ij}S_{ij}}}{|\tau_w|},\ f_r = \tanh\left[\left(1.5 r_1\right)^3\right] \tag{11}$$

3. Result

To evaluate the performance of the progressively corrected SA model, we test it on several representative flow cases. All simulations are performed with a second-order cell-centered finite volume method for the RANS equations. Inviscid fluxes are discretized using the Roe scheme, while viscous fluxes are handled via a second-order central differencing approach. For time integration, an implicit Gauss-Seidel method is utilized.

For a ZPG flat-plate TBL at Re=5×10^6, Ma=0.2, Figure 2 compares the results predicted by the original SA model and the progressively corrected model. As expected, both models closely match the theoretical correlations in the fully turbulent region.

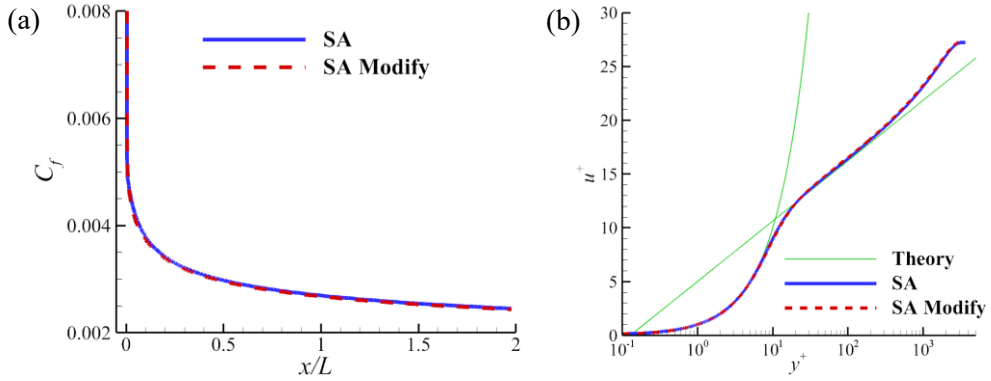

Figure 2 Comparison of results for flat plate. a) Skin friction coefficient, b) Velocity profile at x/L = 0.97

An axisymmetric separated boundary layer[10] at Re=2×10^6, Ma=0.088 is used to further assess APG modeling capabilities. Figure 3 compares the skin friction coefficient and the pressure coefficient. The progressive corrections lead to a noticeable improvement in predicting the extent of separation and reattachment. The near-wall and outer-layer modifications collectively enhance the model's ability to capture APG-driven separation while retaining good agreement in ZPG regions.

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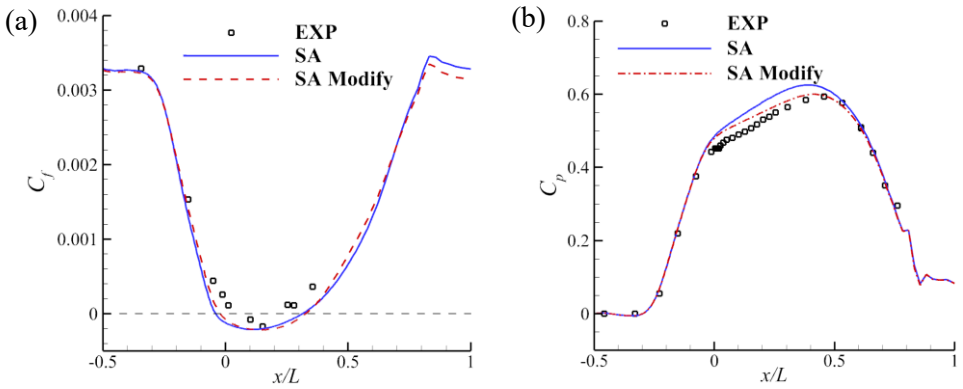

Figure 3 Comparison of results for ASBL. a) Skin friction coefficient, b) Pressure coefficient.

Figure 4 shows the flow around the S809 airfoil[11] at Re=2×10^6, Ma=0.2, AOA=0~20 deg, which experiences significant flow separation at moderate to high angles of attack. The original SA model underpredicts the size of the separated region, whereas the corrected model more accurately captures the pressure plateau in the separated zone. This improvement stems primarily from the modified destruction and production terms tailored for APG conditions, which become pronounced on the suction side of the airfoil at higher angles of attack.

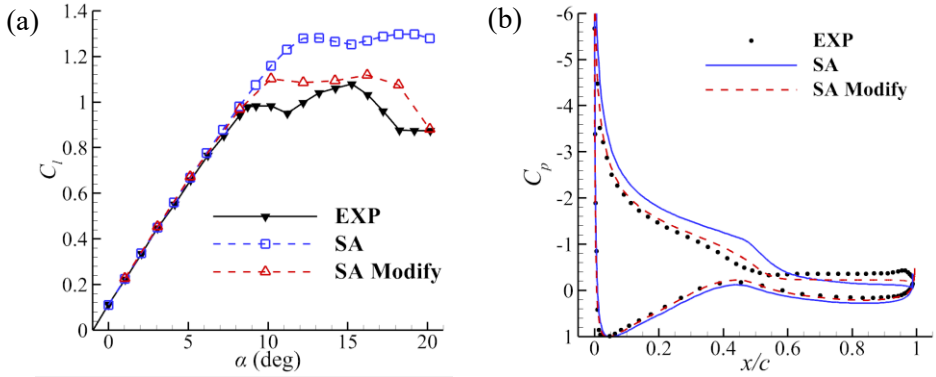

Figure 4 Comparison of results for S809. a) Lift curve, b) Pressure coefficient at AOA=14 deg.

Figure 5 presents the results for separated flow over periodic hill[12] at Re=5600, Ma=0.1. Compared to the original SA model, the progressively corrected model predicts a smaller separation bubble and a more accurate pressure distribution.

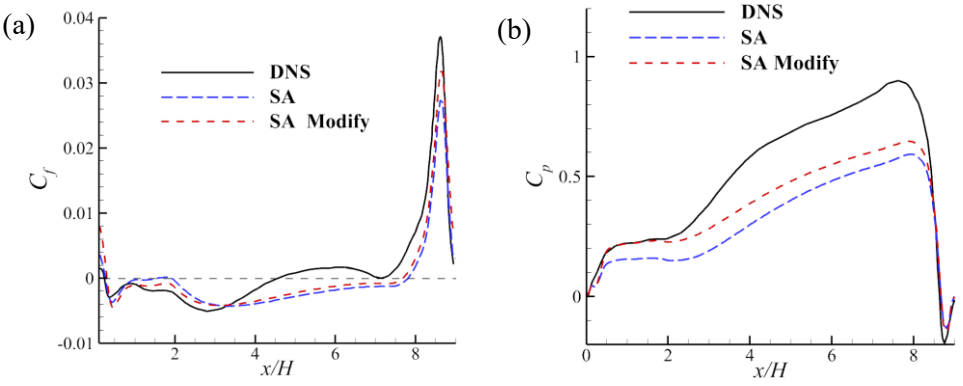

Figure 5 Comparison of results for PH. a) Skin friction coefficient, b) Pressure coefficient.

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4. Conclusion

This paper presents a progressive data-driven framework for improving the Spalart–Allmaras (SA) turbulence model, targeting both near-wall scaling and adverse pressure gradient (APG) effects. By incrementally refining the damping function f_{v1}, the inner-layer destruction term f_w, and the outer-layer production term γ, each correction is introduced without compromising the original calibration. Symbolic regression, informed by high-fidelity LES data of both ZPG and APG turbulent boundary layers, provides explicit algebraic forms that preserve interpretability and promote robust generalization.

Validation on multiple test cases—ranging from flat-plate boundary layers to complex separated flows over airfoils and hills—demonstrates that the progressively corrected SA model achieves higher accuracy in APG flows and separated flows while retaining baseline performance in canonical ZPG flows. Future work will explore further extensions of this progressive framework, such as incorporating compressibility effects and extending the approach to three-dimensional flows with strong secondary motions.

By integrating fundamental turbulence physics with modern data-driven regression, this study offers a blueprint for systematically enhancing RANS turbulence models in a way that balances accuracy, interpretability, and computational efficiency.

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