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Application of multi-fidelity Gaussian Process modelling to the Common Research Model - High-Lift geometry for use in digital environments

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Abstract. This work has investigated the application of multi-fidelity Gaussian Process (GP)-based regression methods to realistic experimental and numerical aerodynamic data. Here the focus was on the Common Research Model (CRM) High-Lift configuration using data generated as part of the High-Lift Prediction Workshop 5. Comparison of coKriging (linear) and non-linear GP methods for the construction of surrogates showed similar levels of accuracy which is attributed to the smoothly varying functions considered. Additionally, the use of proper orthogonal decomposition (POD) combined with the GP-methods allowed for a reduction in the required input points of large data sets by a factor of at least 5.8 allowing GP-methods to tractably handle larger data sets. For most of the points the observed error was less than 10%, however, localised areas of high error were observed the origins of which are to be determined. The integration of the surrogates into the Unity game engine to form a digital twin is shown, with the expected aerodynamic behaviour observed, demonstrating their potential for use in real-time interaction and design-aid.

1. Introduction

Gaussian process (GP)-based regression methods are a powerful mathematical tool allowing incorporation of multi-fidelity method (MFM) data to improve interpolative accuracy. However, these methods have seen infrequent testing on realistic aerospace data sets which can suffer from simultaneously partially-sampled and large data sets, which can be both scalar or tensorial.

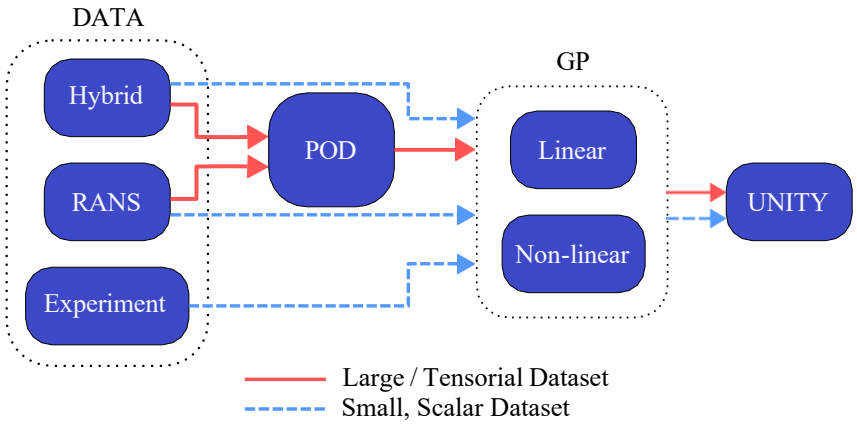

Figure 1: Workflow of the Gaussian Process-based surrogate modelling and digital-twin construction.

The poor scalability of GP-based methods with training sample size stands as a significant obstacle to the use of exact GP methods with large data sets, such as field data like velocity or skin-friction in a fluid dynamics context. Additionally, tensorial data, unlike simple bulk quantities such as force and moment coefficients, contains inherent, systematic correlations within the data that are not typically captured in

simple single-variate GP methods. Methods such as multi-variate GP methods can be used to handle this issue but can be challenging to implement. An alternative method, among many, that can ameliorate this issue is using low-dimensional representation such as proper orthogonal decomposition (POD).

One application for surrogate models is use in digital environments and digital twins for data exploration and design analysis which has expanded over the past years. These digital concepts typically make use of some form of neural-network or reduced-order model. However, by contrast, the potential use of GP-based surrogate models in virtual environments is relatively unexplored. Figure 1 shows a summary of the workflow for this study indicating the different fidelity data used, how it will be treated using GP-based regression and POD, and then implemented into the Unity game engine. The specific details of this will be discussed in the following sections.

2. Methodology

The GP-based regression methods used in this study were all run using in-house codes that make use of the GPy library developed at Sheffield University [1]. In all cases the Matern52 kernel has been used to construct the co-variance matrix and the BFGS optimisation algorithm was used with 5 optimisation restarts. The standard linear GP (coKriging) method and the non-linear (Deep) GP method available in GPy were used. The POD was likewise done using the standard libraries from Numpy.

2.1 High Lift Prediction Workshop (HLPW) Dataset

The HLPW [2][3] is a series of workshops looking at the high-lift variant of the Common Research Model (CRM) standardised aeroplane geometry [4]. Participants are asked to run a number of cases with different points of interest in mind using a range of codes and modelling techniques. Experimental data is also available from full-model wind tunnel tests as seen in figure 2.

For this study we focused on Case 2.2 which corresponds to a specific geometry at a Reynolds number of 5.9×10^6 and a Mach number of 0.2 operating at different angles of attack (AoAs). For seven angles the participants submitted the requested data including force and moment coefficients, wall shear stress components and pressure coefficient. To conform with the request of the workshop where participant data has been used reference is made to it using the pseudonymised references utilised by the workshop. All of the data in this case came from participant R-006.

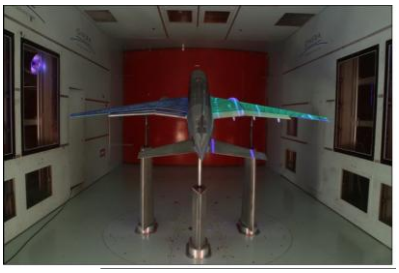

Figure 2: Full CRM-model in wind-tunnel experiment from HLPW [2].

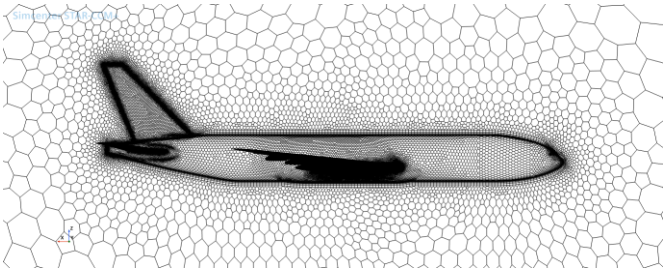

Figure 3: Computational coarse surface and volume mesh used for RANS simulations.

2.2 Further RANS simulations

Subsequent Reynolds-Averaged Navier-Stokes (RANS) simulations were run replicating the method of [4], with the SST $k - \omega$ turbulence model. To emphasise the low/high fidelity nature of the data the coarsest mesh of the 2.R.04 mesh series, seen in figure 3, consisting of 17.7 million cells was used. Additionally, rather than rigorous convergence criteria, each AoA was run for 1000 iterations. Understanding of the force data obtained from the CFD results can be gained from figure 4. As expected the

data follows the rough trend correctly but is inaccurate quantitatively for most of the range of angles.

2.3 Unity Game Engine

The Unity game engine is a physics engine developed for use in game development, but which may also be used to construct digital twins. The flexible capabilities of the engine allows one to use user-written scripts to apply forces and moments to rigid bodies that can be based on imported STL files. In-house developments at the University of Manchester include virtual wind tunnel elements for flow visualisation such as streamlines, introduced particles and vortex filaments. Current methods for calculating forces rely on simple mathematical models using a panel method that blends with Newton's sine-squared law at an arbitrarily chosen stall angle which can be run in real-time but does not incorporate CFD data.

3. Results

In this section we present the most significant results from each of the study components considered in the introduction and figure 1.

3.1 Force & Moment Surrogate Construction

Figure 4 shows the produced surrogate models for the mean drag and lift coefficients discussed with the associated uncertainty for the coKriging and non-linear GP methods with different numbers of randomly drawn sample points used as input (pitching moment coefficient surrogates were also constructed but are omitted here). It is seen that due to the smooth nature of the functions few data points are required to get a reasonable approximation of the experimental curve and the incorporation of a few high-fidelity samples offers a substantial improvement over a single low-fidelity model (green curve). This is especially true of the lift curve that is nearly linear within the range. The expected trend of reducing error and uncertainty with increasing low- or high-fidelity input data is clearly observed.

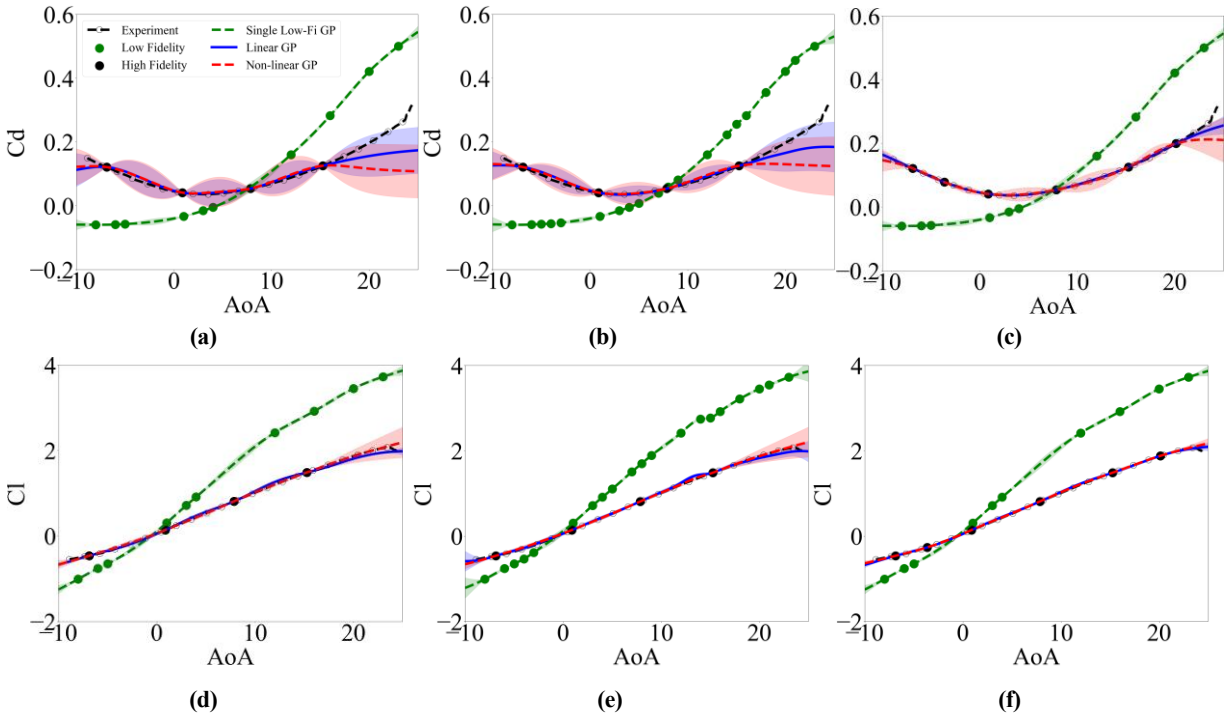

Figure 4: Surrogate models for the force coefficients C_D (top) and C_l (bottom) generated using the two GP methods with shaded area indicating uncertainty using different low- and high-fidelity sample sizes. Number of input points: (left) $n_h = 4, n_l = 10$, (middle) $n_h = 4, n_l = 20$, (right) $n_h = 6, n_l = 10$.

Similar conclusions can be made by considering the *L*2-norm error based on the experimental data, ignoring uncertainty in the data, as a function of number of low- and high- fidelity input points (for $n_{low} \geq n_{high}$) shown in figure 5. A similar degree of error is seen for all the surrogates, though consistent with the above the linear model performs slightly better, though the difference rapidly diminishes as n_h and n_l increase. The reduction of error with increasing input points, especially for the high-fidelity, is seen. The use of random sampling introduces a substantial degree of noise here due to changing the extent of extrapolation versus interpolation being done by the surrogate since the end points change at random intervals. Comparing the maximal variance and error we observe a degree of the expected correlation, since hopefully the maximum error corresponds to the maximally uncertain points, though with a degree of obscuring noise caused by the random sampling as discussed.

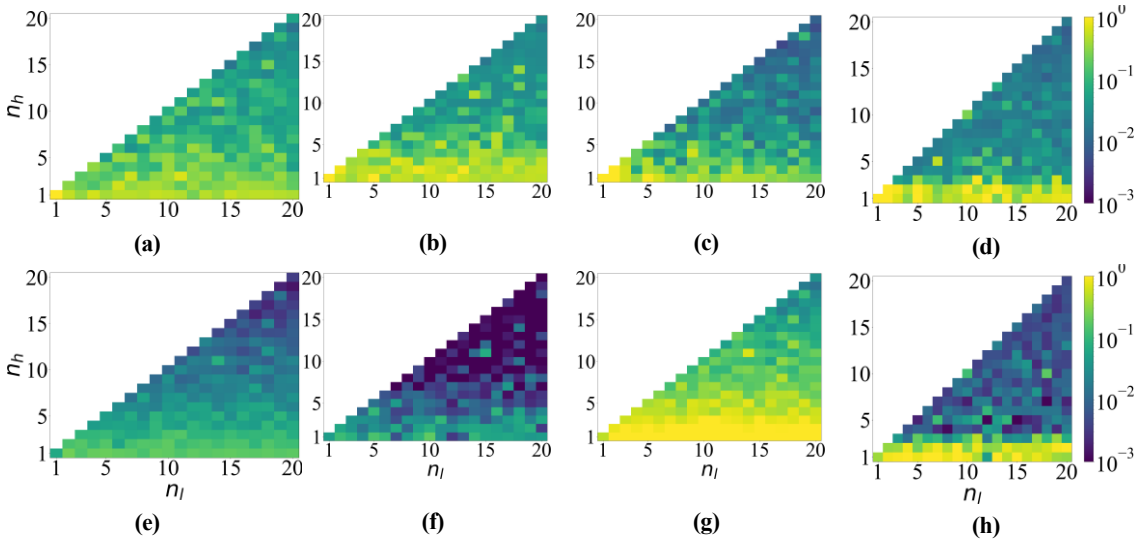

Figure 5: Normalised *L*2-norm error (top) and maximal variance (bottom) for drag and lift surrogate predictions against the number of low-fidelity samples, n_l, and high-fidelity samples, n_h used as input. (a,e) C_D with Linear model, (b,f) C_D with Non-linear model, (c,g) C_L with Linear model and (d,h) C_L with Non-linear model.

Further work will consider more closely the effect of kernel choice, sampling method and the method of optimisation of the hyperparameters on the prediction error.

3.2 Handling Large Data using POD-GP Surrogate Model

We turn now to the proposed method of handling large or tensorial data. By applying POD to the pressure coefficient line data measured over the wing surface, as shown in figure 6a, a reduced-order representation of the field could be constructed. By then projecting either the full set or truncated set of modes onto a common basis set - a chosen AoA's modes - a set of up to 20 modes per slice that only vary in amplitude between the AoAs was constructed. With the modes using the common basis functions a single-fidelity GP model for each mode amplitude was constructed using the linear model. In this way the full set of data of 7×2323 points was reduced to 7×400 at most, a reduction of a factor of 5.8.

In order to validate the constructed POD-GP surrogate model for the pressure coefficient the POD-GP model was trained using six of the seven available data points, with the first AoA of 6 deg, used as the basis. The model was then used to make a prediction at the omitted point and compared with the known data. This is shown in figure 6 for an AoA of 20 deg which shows the model prediction and the error taking the RANS case as the base truth. For the vast majority of the points the error is lower than 10% but for some of the rows there are highly localised areas of high error up to 100%. The origin of this

error has yet to be determined and remains under investigation. One potential source of error may be in the choice of angle for the common basis.

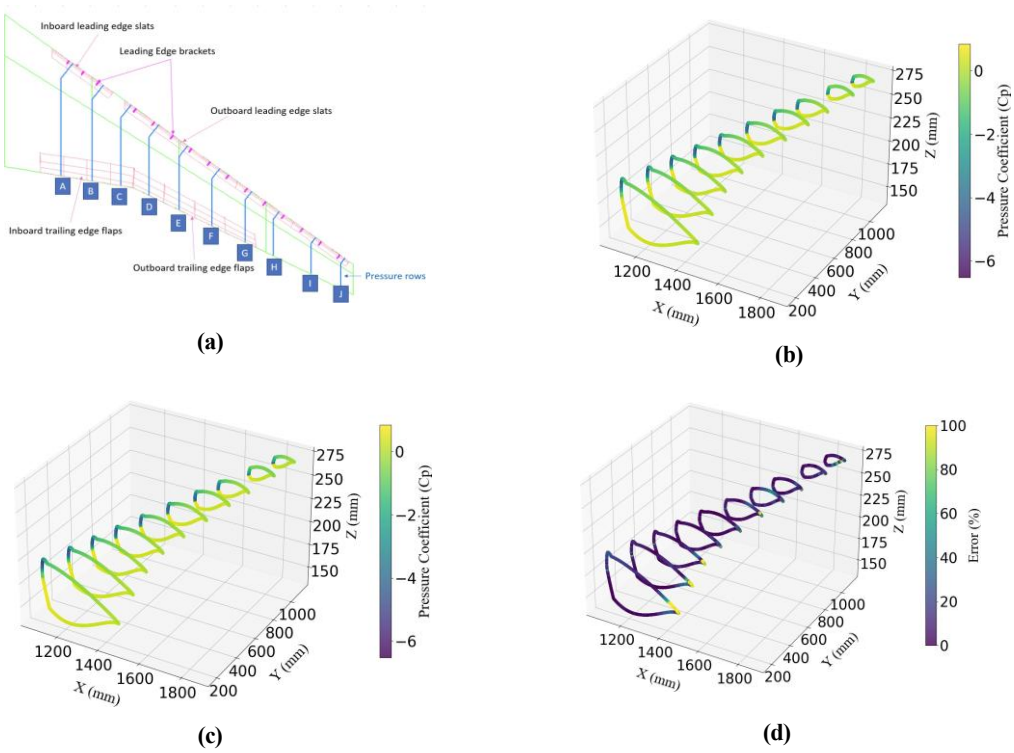

Figure 6: (a) Pressure measurement physical locations from the workshop data [2], (b) POD-GP reconstruction compared with (c) RANS data for AoA = 20 deg and (d) percentage error between them.

This method has been applied to strips of data given they were already available but the method can easily be extended to full surfaces. Further work will explore using Lagrangian interpolation to interpolate over the surface from the sparse data points and also extend the method to properly tensorial data such as the skin friction. Extension to multi-fidelity data, incorporating hybrid RANS/LES, is also to be explored.

3.3 Implementation into Unity Engine

To explore the possibilities of use for the generated surrogates they were subsequently ported into the Unity game engine which has been discussed above. By attaching the surrogate model to a geometric representation of the CRM geometry a digital twin of the system was generated. Figure 7 shows the digital twin of the plane with a constant thrust applied and the surrogate models providing the drag, lift and pitching moments as a function of the angle of attack of the plane where the forces are illustrated by coloured vectors. Though not visible in still image, the animation of the aeroplane shows the expected phugoid motion given the absence of any pilot control or moveable control surfaces.

Figure 8 illustrates a comparison with the currently utilised panel/sine-squared method. We can see that the panel method provides a reasonable approximation of the lift curve produced by the MFM surrogate though it underestimates the peak lift coefficient. On the other hand, we see that the GP method goes to zero very quickly in extrapolation. One possibility to be explored is to combine the two methods. The surrogate could be used as the determinant of the location of stall for switching between the panel and sine-squared law or could likewise use the sine-squared law outside the domain of the input data.

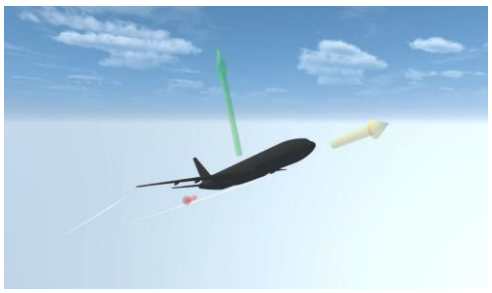

Figure 7: Unity implementation showing plane in flight using MFM surrogates for drag, lift and pitching moment.

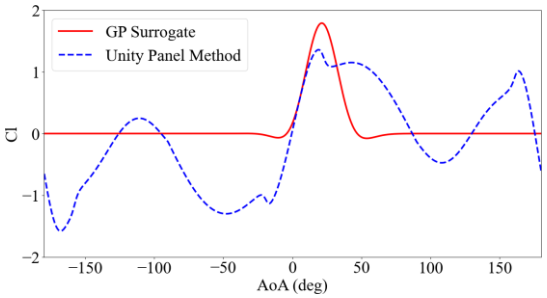

Figure 8: Comparison of lift coefficient C_L from surrogate model against current in-house unity implementation using panel/sine-squared method.

4. Conclusions

The study presented here has considered the relative merits of different GP-based methods for constructing multi-fidelity surrogate models based on realistic aerospace data. Surrogates have been constructed for force and moment coefficients using multi-fidelity coKriging and non-linear GP methods. The correct trends with sensitivity of accuracy to number of sample points have been observed, with both methods showing comparable levels of accuracy owing to the smoothly varying curves.

It has also been demonstrated using pressure coefficient data from RANS simulations that GP regression applied to POD modes projected on a common basis set is accurate for most of the wing, though with some localised areas of high error the origin of which is to be determined. In general, this allows efficient handling of larger data sets that would otherwise be intractable using GP methods, with a reduction of at least a factor of 5.8 made here. Further work will consider expanding to tensorial data and methods for interpolating over the whole wing surface, as well as the comparative behaviour of different GP methods.

Finally, the feasibility of using GP-based surrogates for digital twin construction has been demonstrated by integration with the Unity game engine, with the expected aerodynamic behaviour observed. Improvement over the current panel based methods have been discussed though limitations, especially in extrapolation, remain for the GP methods and proposed future work has been indicated.

Acknowledgments

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