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Finite Element Neural Network Method for Navier-Stokes Equations

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Abstract

This work focuses on the modeling of the incompressible Navier-Stokes equations with the Finite Element Neural Network Method (FENNM), a hybrid approach where Physics-Informed Neural Networks (PINNs) are enhanced by the Finite Element Method (FEM). PINNs build upon a neural network which acts as a solver harnessing physical laws through additional loss functions. They are universal function approximators and meshless, by using automatic differentiation to compute derivatives for Partial Differential Equations (PDEs). PINNs have been applied as forward and inverse solvers to cases ranging from structural to fluid mechanics, demonstrating their flexibility and versatility. However, they require a tedious tailoring to converge during training, as they are a recent approach that does not yet benefit from a systematic implementation process. Herewith, this work features the integration of FEM in the PINNs framework, to evolve from a collocation method to a weighted residual one, to infer a correlation between the collocation points. While PINNs integrate the strong form of PDEs, we use a Petrov-Galerkin approach where the product of the NN output as the trial solution with Lagrange test functions is integrated with a Gauss-Legendre quadrature rule. Residuals are evaluated at quadrature points in elements constituting a uniform mesh. Defining test functions and quadrature weights as filters, convolution is used to compute parallelly the losses per element and test function. The FENNM has been applied to the 2D cases of the lid-driven cavity flow, Rider-Kothe vortex and flow past a square, yielding better accuracy than PINNs for similar architectures and number of collocation points. FENNM shifts the complexity from the training to the definition of the losses, ensuring convergence once implementation is adequate.

Keywords: incompressible flows, FEM, PINN, FENNM