Asymptotic Model for Flow in Heterogeneous Media

[1]Hamid Ait Abderrahmane*; [2] Moussa Tembely

[1]Khalifa University Abu Dhabi, United Arab Emirates

[2]Concordia University, Quebec, Canada

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Abstract.

Current methods for modelling subsurface flow primarily rely on the Darcy equation. Although this equation is widely used, it can oversimplify fluid flow in porous media, resulting in discrepancies between numerical simulations and experimental observations. This study aims to identify the partial differential equation (PDE) that accurately describes fluid flow in heterogeneous porous media by utilizing a data-driven modelling approach. We began with the classic problem of flow over a cylinder in a channel and gradually increased the complexity by adding multiple cylinders and constructing configurations that simulate heterogeneous porous media. Using sparse regression algorithms, we inferred a reduced PDE model for each configuration based on spatiotemporal snapshots of the vorticity field, which were derived from the continuity and Navier-Stokes equations. Our results indicate that the convective terms in the vorticity transport equation diminish, while quadratic terms become more prominent. Furthermore, as the flow configurations more closely resemble a heterogeneous medium, the coefficients of the discovered PDEs exhibit time dependency, showing greater regularity and periodic behaviour. In conclusion, this study lays the groundwork for refining the Darcy equation as an asymptotic limit for fluid flow in complex porous media. It also raises questions about the equation's ability to accurately represent the underlying physics that govern the growth of small perturbations in hydrodynamic instabilities.

1. Introduction

Recently, the discovery of physical principles in dynamical systems through data-driven methods has emerged as a vibrant area of research, resulting in a variety of proposed techniques. One notable method is the Sparse Identification of Nonlinear Dynamics (SINDy), which has proven effective in rediscovering the Navier-Stokes equations using simulation data from well-established fluid dynamics problems. This approach utilizes sparse regression to identify partial differential equations (PDEs), as demonstrated by S. H. Rudy and colleagues in 2017.

The central concept behind SINDy is to construct a dictionary of simple functions along with their corresponding spatial partial derivatives that may characterize the dynamics of the dependent variable. Sparsity-boosting techniques are then applied to select the most relevant candidates that accurately represent the system's dynamics. Rudy et al. (2017) successfully applied SINDy to identify PDEs in several cases, including the Navier-Stokes equations, the quantum harmonic oscillator, and the diffusion equation. They also expanded their research to include the identification of PDEs with non-constant coefficients (Rudy et al., 2019).

Unlike canonical flow dynamics problems, such as flow around circular cylinders and cavities that are often used to test data-driven model algorithms, flow through porous media is more complex and exhibits intricate flow dynamics. While several studies have successfully applied various PDE identification methods to canonical models (Hasan et al., 2020; Long et al., 2019; Zhang & Liu, 2021), there are few studies that focus specifically on fluid flow in heterogeneous porous media.

In this study, we used the open-source PDE-FIND algorithm (S. Rudy et al., 2019) to identify the underlying partial differential equations (PDEs) governing fluid flow in various geometries. We specifically created a complex geometry to simulate porous media and to better understand the flow dynamics within heterogeneous reservoir rocks. The identified PDEs could enhance our understanding of flow dynamics in complex porous media. This advancement can help address reservoir engineers' challenges, particularly the history matching problem, which partially arises from the simplified flow model (the Darcy equation) used in reservoir simulators.

2. Formulation of the problem

The simulations were conducted with a Reynolds number of 100 in a complex geometry. The Reynolds number decreases within the complex geometry because the fluid velocity within the porous media is lower. The Reynolds number is calculated by.

$$Re = \frac{\rho u L}{\mu}$$

where Re is the Reynolds number, ρ is the fluid density, u is the flow speed, L is the characteristic linear dimension, and μ is the dynamic viscosity of the fluid. The data were generated by solving two-dimensional fluid flows across various geometries, ranging from simple to complex. The Navier-Stokes equations were solved using the Immersed Boundary Projection Method (IBPM), as outlined by Colonius and Taira (2008) and Taira and Colonius (2007). We considered six different geometries, which included cylinders of varying diameters and positions. These cylinders were arranged to create a more complex geometry that mimicked the structures commonly found in porous media, as illustrated in Figure 1.

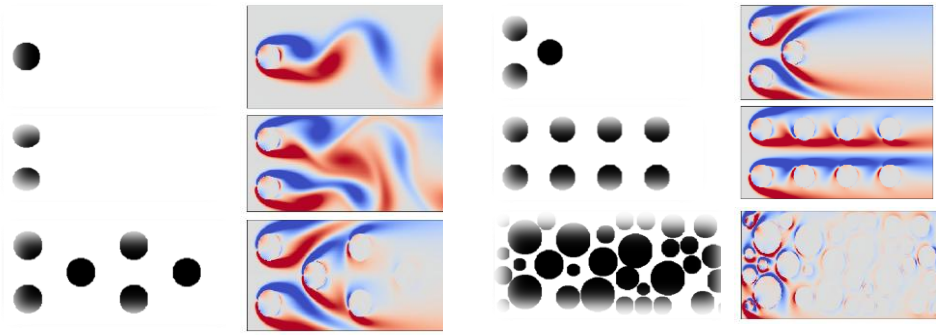

Figure 1. Six geometries with increasing complexity and a snapshots of vorticity fields of the flow through the six geometries

For each case, the physical dimensions are 9 by 4, and the grid resolution is 449 by 199. The flow solver uses a third-order Runge-Kutta integration scheme with a time step of $\Delta t = 0.02$. A total of 1,000 snapshots have been collected.

3. Methodology

The open-source PDE-FIND algorithm (Rudy et al., 2019) was utilized in this study to discover the governing equation based on time-series snapshots formatted as follows:

$$u_t = N(u, u_x, u_{xx}, \dots, x, \mu)$$

The subscripts indicate partial differentiation and represent the controlling parameters within the system N. This function depends on various spatial derivatives of the variable we aim to discover. The

algorithm serves to identify the dominant term from a library of candidate terms that includes the spatial derivatives of the dependent variable u and their combinations. There are no restrictions on these polynomial combinations of partial derivatives, allowing for nonlinearities. This library is denoted as $\Theta(U, Q)$. The column vector Q contains any additional information that can be combined with (for instance, Reynolds numbers) to form a matrix. By taking the time derivative u_t and reshaping it into a column vector, we can derive a linear equation that represents the target partial differential equations (PDEs) as follows:

$$U_t = \Theta(U, Q)\xi$$

A sparse vector ξ of coefficients is obtained using sparse regression methods (e.g., Ridge). The goal is to develop a simple model that captures the intricate dynamics hidden in the spatial-temporal evolution of the system. Once these coefficients are determined, certain terms in the matrix will be "activated." The sum of the products of these active terms and their corresponding coefficients will yield the partial differential equations (PDEs) for the cases being studied. The workflow involves reshaping the vorticity field into a column vector and arranging these side by side to create a matrix of successive snapshots. Given the complex geometry we are dealing with, we need to subsample the data, meaning we will only collect data from regions where there is fluid present. To achieve this, we must design a measurement matrix that extracts the values of the vorticity field while excluding the solid matrix of the medium.
We create the library matrix using the spatial derivatives calculated through the finite difference method. Then, we employ a sparse regression method to identify the significant active terms within the library and to derive the most straightforward partial differential equation (PDE) that accurately represents the dynamics of the vorticity field.

4. Results and Discussions

Figure 1 displays the flow simulation results for selected cases. Because the fluid domain is not continuous, we collected data on vorticity at randomly selected points throughout the fluid domain. These points can be thought of as sensors distributed randomly across the medium. The results are not influenced by the distribution of the sensors but rather by their quantity. Therefore, the number of randomly distributed sensors is increased until the driven partial differential equation (PDE) becomes consistent. The algorithm enables the coefficients of the data-driven PDEs to evolve; these equations are presented in three cases shown in Figure 2.

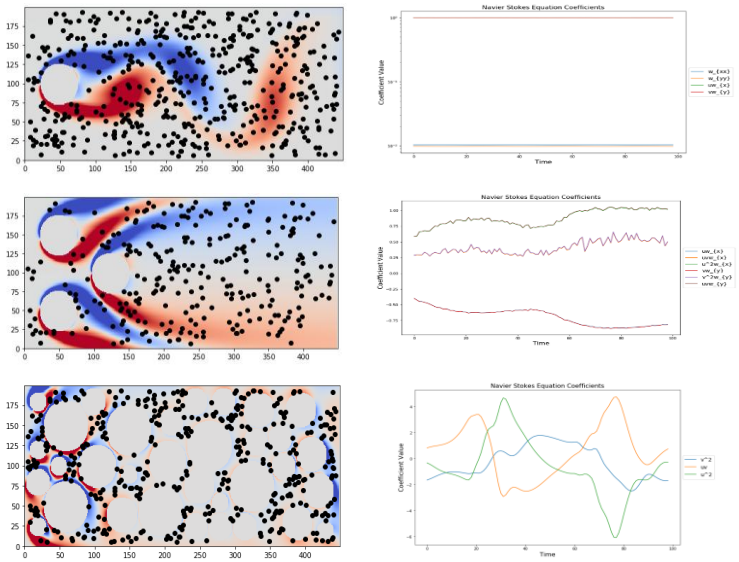

Figure 2 Model of vorticity for three selected geometries

We noticed that from three cylinders the coefficients of the PDE start to depend on time. From three cylinders, the fluid seems to become unstable. On can notice that as the number of the cylinder increases, the velocity between the cylinders (pores and throats) decreases resulting in the decrease of the Reynolds number. The diffusion term in data driven PDE become stronger in addition the nonlinear term become of higher order. The study indicates as the number of the cylinder increases and the media become closer to porous media the asymptotic equation to the Navier-Stokes equation is not Darcy equation but a nonlinear equation with strong nonlinearities.

5. Conclusion

In this study, we examined partial differential equations (PDEs) under various geometric conditions, with a particular emphasis on flow dynamics within porous media. Our findings indicate that we can identify appropriate PDEs under simple geometric conditions, while more complex geometries result in PDEs with variable coefficients. Ultimately, the PDEs identified in the context of porous media are presented in a more simplified form, which underscores the potential for accurately deriving modified or non-linear alternative equation to Darcy's law, a fundamental principle governing fluid flow through porous materials.

6. Acknowledgments

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