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Physics-informed local-global underground fluid flow modeling with multiple sinks

G M Shutov[1], **D I Akhmetov**[1], **E V Burnaev**[1,2], and **V V Vanovskiy**[1]

[1]Center of Artificial Intelligence, Skolkovo Institute of Science and Technology, Moscow, Russia
[2]Autonomous Non-Profit Organization Artificial Intelligence Research Institute, Moscow, Russia

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Abstract. In the study we present a novel approach to predict pressure field in a geological fluid reservoir in case of slightly-compressible single phase unsteady-state flow and highly-heterogeneous permeability field in 2D. The key feature of our approach is that it effectively solves a global problem with point sinks (production wells) which interfere with each other by using a local single-well solution. The local single-well model is a physics-informed neural network that demonstrates solid performance: mean absolute percentage error (MAPE) in estimation of pressure field around a well equals to 1.21 %. The global problem of the whole reservoir simulation with multiple sinks is reduced to the solution of inverse problem – finding such pressure field for the whole reservoir that is consistent with given permeability field, bottomhole well pressures and well production rates and also with local problem solutions for each well. To solve this problem, we implement an iterative optimization algorithm that uses the trained estimator from the local problem leveraging the neural network fast inference. The resulting pressure fields after the optimization are very close to the ground truth pressures. At the same time the estimated production rates diverge from numerical simulator production rates, however keeping correct trend.

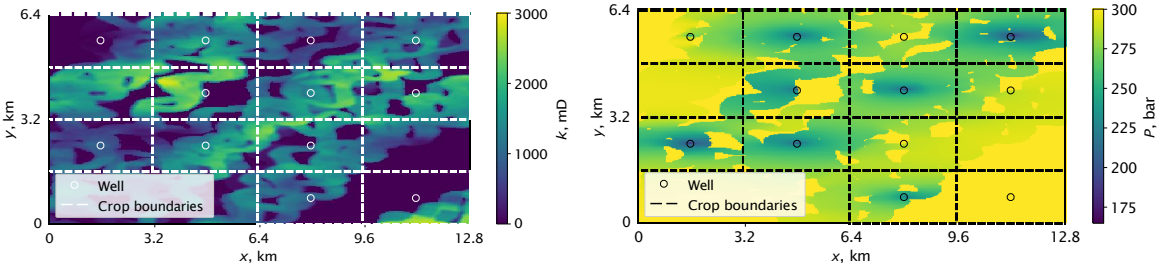
1 Introduction

Conventional method of flow simulation in geological reservoirs is a numerical solution of differential equations []. Such methods can provide sufficient prediction accuracy, but the approach is time-consuming and computationally demanding, especially for inverse problem solving (e.g. history matching in reservoir engineering) where usually multiple simulation is needed. As an alternative, the application of neural network proxy-models for acceleration of hydrodynamic simulation has been the subject of intensive research nowadays []. One of the promising neural network approaches that takes into account the governed differential equations is called 'physics-informed machine learning' (PIML) []. A distinctive feature of the methodology is a loss function that directly incorporates the physics equations. The approach limits the solution space of a neural network increasing the reliability of predictions. However, existing works that apply machine learning to simulation of fluid flow in porous media does not take into account the specifics of flow around point sinks (e.g. wells in subsurface reservoir) and the sinks interference. The problem is of great importance in the domain of reservoir engineering, where a reservoir is depleted with a network of wells. In the framework of the research we develop a physics-informed neural network to solve a local problem of determining the pressure dynamics and flows around a single well. Then the local problem solution is used to solve a global problem of pressure field determination which is formulated as an inverse problem of finding the appropriate unsteady boundary condition for each single-well problem. The approach allows one to simulate the physics of fluid movement in the presence of the wells what makes it applicable for simulating real subsurface reservoirs.

2 Materials and Methods

2.1 Dataset Generation

We consider 2D problem, where each computational scenario has unique permeability spatial distribution $K(x, y)$. The permeability maps, utilized for dataset generation, correspond to Brugge benchmark []. Each permeability field is highly-heterogeneous having many so-called fluvial channels. Figure shows an example of such permeability map.



(a) (b)

Figure 1: Random scenario in generated dataset: demonstrates the heterogeneous permeability field with 12 wells. shows pressure field in the second simulation step calculated using the permeability map. It is well seen that the pressure remains the same in impermeable zones. Dashed line outlines the sections with well in the center denoted by circles. We use these sections for neural network training.

We use a commercial numerical simulator to generate the dataset. The solver utilizes the Peaceman formula [] for sink/source terms approximation. We use this approximation in the inverse problem setting to calculate production rates. In each computational scenario, the numerical model is initialized with reservoir pressure P_{init} and an arbitrary number of production wells (point sinks) equal to N_w located in the computational domain of size (X, Y). The wells start to operate at the initial time point with constant bottomhole pressure p_{bhp} chosen randomly from an uniform distribution between p_{bhp}^{min} and p_{bhp}^{max}]. Each simulation continued for 21 time steps T, and at each time step the pressure field P, well production rates q and botomhole pressures are saved. As a result, a dataset of permeability fields K of shape (N, X, Y) and their corresponding pressures P of shape (N, T, X, Y) was generated, where $N = 3450$ is the total number of simulations. Table shows the quantitative properties of the generated dataset.

Table 1: Properties of generated dataset.

Model properties	Values
Number of simulations	3450
Number of well sections	20096
Number of cells in a reservoir, XYZ	256 × 256 x 1
Number of cells in a well section, XYZ	64 × 64 x 1
Reservoir size, m	12800 × 6400 x 2
Well section size, m	3200 × 1600 x 1
Number of wells in a scenario	1-16
Initial reservoir pressure, bars	300
Permeability range, mD	0 - 3922.25
Simulation duration, month	60
Number of time steps	21

Our local model works on a well-by-well basis, i.e. the model input is not the whole considered reservoir with shape (X, Y), but the section with shape (x, y) with a well placed in the center. We

crop each spatial domain of shape (X, Y) from the generated dataset into sections with shape (N_w, x, y) where N_w is the number of wells in a scenario as shown in Figure. Note that P, K, X, Y stand for the whole reservoir's pressure, permeability and coordinates and small letters p, k, x, y stand for the same variables for a single section.

2.2 Forward problem solution

2.2.1 Neural network

We train a UNET-like neural network [] which is often used in physics-informed machine learning tasks on spatial domains [,,]. The model takes as input permeability map k, a constant well bottomhole pressure p_{bhp} and unsteady boundary condition $p_{bc}(t)$ which is the pressure at the section boundaries. The input tensor shape is $(bs, T+1, x, y)$, where bs - batch size; T outlines the time steps of the unsteady boundary condition; $+1$ dimension denotes stacked permeability map k; x and y are spatial dimensions of the section. All the T tensors of the input corresponding to T time steps contain pressure values in the well and at the boundaries. Bottomhole well pressure value p_{bhp} is placed in the center of the section and is the same for all time steps. The boundaries of the tensor are filled with the boundary conditions $p_{bc}(t)$. The area between the well and the section boundaries is filled with zeros. The output of the neural network is the pressure field in a section at each time step p of shape (T, x, y). We divide the full dataset into three splits: train-validation-test sets in the proportion of 80 %, 10%, 10% for training, hyperparameters selection and validation.

2.2.2 Physics-informed loss

To preserve the physical realism of the neural network solutions, we suggest physics-informed loss function consisting of three components: L_{data}, L_{PDE} L_{bc}. L_{data} is data loss component which can be expressed as:

$$\mathsf{L}_{data} = ||\hat{p}(x, y, t) - p_{data}(x, y, t)||_2^2, \tag{1}$$

here $\hat{p}$ is the pressure field predicted by the neural network with weights θ, and p_{data} is the pressure field calculated in the numerical simulator. The brackets $||\cdot|_2^2$ denote the squared second norm. The data loss function is responsible for matching the predicted pressures $\hat{p}$ to the calculated p_{data} pressure fields. The second component of the physics-informed loss function is L_{PDE}, which is the squared residual between the left and right parts of continuity equation for flow in porous media:

$$\mathsf{L}_{PDE} = ||\phi \frac{\partial(\rho(\hat{p}))}{\partial t} - div\,[\rho(\hat{p})u(k, \nabla p)] - q||_2^2, \tag{2}$$

where $\rho(\hat{p})$ is the fluid density obtained from the predicted pressure field $\hat{p}$, ϕ is reservoir porosity, $\partial/\partial t$ – time partial derivative, div is divergence operator, $u(k, \nabla p)$ – the flow velocity which is calculated using Darcy equation, q – point sink term. Minimization of this loss component helps to simulate the physics of fluid flow in the neural network predictions. The third component of our loss L_{bc} stands for the difference between the unsteady boundary condition $\hat{p}_{bc}$ which the network predicts and the boundary condition p_{bc}^{data} from the input:

$$\mathsf{L}_{bc} = ||\hat{p}_{bc} - p_{bc}^{data}||_2^2, \tag{3}$$

The loss component is introduced to have more control on prediction of the boundary condition during training.

The overall error function for the neural network training could be expressed as:

$$\mathsf{L} = \mathsf{L}_{data} + \lambda_1 \mathsf{L}_{PDE} + \lambda_2 \mathsf{L}_{bc}, \tag{4}$$

where λ_1 and λ_2 are hyperparameters that were selected using Bayesian optimization [].

2.3 Global problem solution

The global problem of simulating the whole reservoir could be reduced to local ones around each well if we would know the pressure boundary conditions for all the N_w local problems. Taking into account that the unsteady boundary conditions p_{bc} are unknown but should correspond to whole pressure field P, naturally, the inverse problem arises. It could be formulated as finding such pressure boundary conditions p_{bc} for all the wells that correspond to given permeability fields k derived from whole field K, bottomhole pressures p_{bhp}, production rates q and also to the global pressure field P obtained by joining single-wells solutions p. In such a way global and local problems are linked via pressure for every local single-well problem that should be a corresponding crop of the global pressure field.

We propose the boundary conditions p_{bc} optimization procedure leveraging the fast inference of the trained neural network from Section. To obtain well production rates during optimization $\hat{q}(\hat{p})$ we calculate them by means of Peaceman approximation using the pressure field predicted with the neural network. We call the optimization approach "local adaptation" because it works with not a full reservoir but with wells and its interior region separately. The algorithm shows the pseudocode of the approach. The update of boundary condition is performed utilizing weighted average of the form:

$$p_{bc}^{i+1} = \frac{p_{bc}^{i} + \alpha \hat{p}_{bc}^{i}}{1 + \alpha}, \tag{5}$$

where p_{bc}^{i+1}, p_{bc}^{i} - new and current step boundary conditions, $\hat{p}_{bc}^{i}$ - boundary conditions predicted by neural network, α - learning rate which is hyperparameter, i - iteration step.

Algorithm 1 Pressure Boundary Condition Optimization

Initialize pressure field guess $P_{init}(X, Y, T)$ for the full reservoir.
Crop pressure boundary condition p_{bc}^{0} around each well j from $P_{init}(X, Y, T)$.
Optimize p_{bc}:
for i in $1, \ldots, n_{\text{iter}}$ **do**
    Predict pressure around wells $\hat{p}^{i}$ using neural network.
    Calculate production rate q_j^{pred} using Peaceman approximation for each well j.
    Calculate mean squared error: MSE(q_j^{pred}, q_j^{true}) where q_{true} is production rate from the numerical simulator.
    if $|q_j^{pred} - q_j^{true}| < \epsilon$ **then**
        break
    end if
    Renew p_{bc}^{i} using equation.
end for

3 Results

3.1 Forward problem

We validate the performance of the neural network model on the test set and obtain mean absolute percentage error (MAPE) which is equal to 1.21 % and root mean squared error (RMSE) which is equal to 3.8 *bar* between the pressure field from numerical simulation p_{data} and predicted pressure field $\hat{p}$. Figure shows the neural network performance.

3.2 Global problem

Boundary condition p_{bc} optimization via algorithm results in physically realistic pressure fields, however, quite a big divergence between estimated and true production rates - mean absolute percentage error

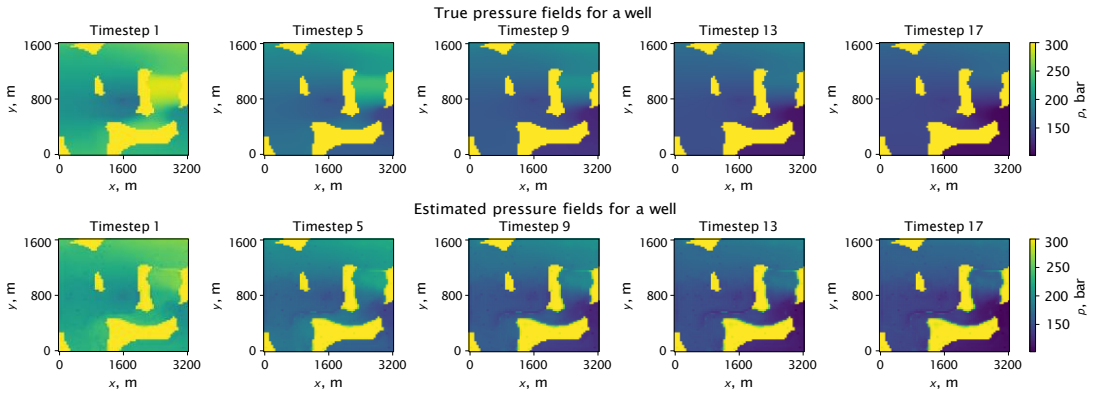


Figure 2: Pressure fields estimated with neural network $\hat{p}$ (upper row) and the ground truth pressure fields p (lower row) for a random well section from test set.

is equal to 50 %. Figure outlines comparison of the ground truth pressure fields p_{data} and the pressure fields obtained by means of the optimization procedure $p\hat{}$. Figure demonstrates the resulting production rates from optimization and the ground truth production rates.

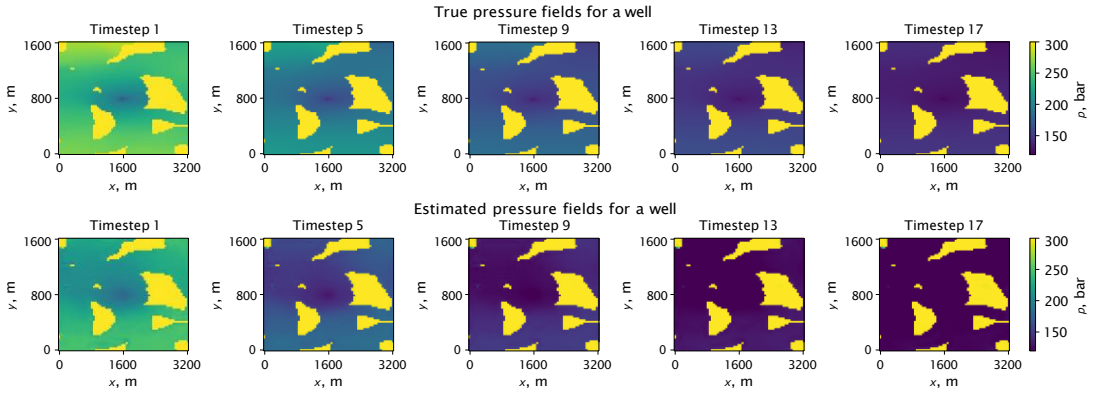


Figure 3: Resulting pressure fields $\hat{p}$ for a well section utilizing the local adaptation optimization strategy. The upper row of pictures - numerical simulator output and the lower row - neural network estimation after boundary condition optimization.

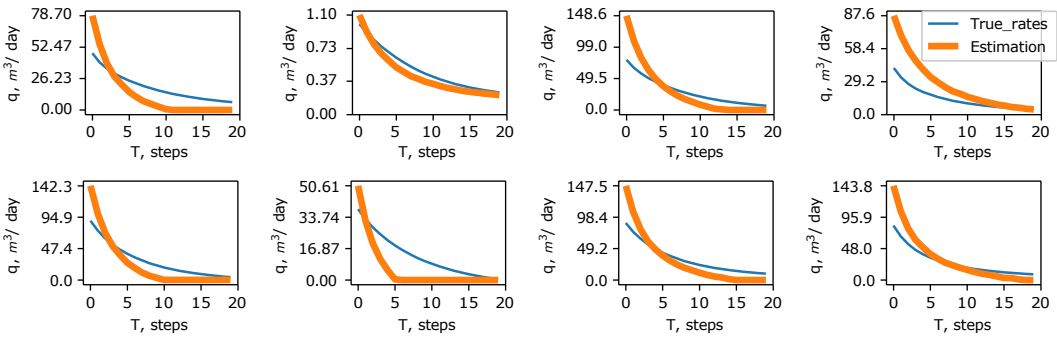


Figure 4: Well production rates from numerical simulator (blue line) and production rates calculated after the optimization of boundary conditions(orange line).

4 Conclusion

We propose a novel local-global approach to estimate pressure field in a geological fluid reservoir with multiple sinks. For this we solve a local problem of pressure evolution around a single sink source (production well) via physics-informed neural network model. The experiments show that the local model works with mean absolute percentage error (MAPE) of 1.21 %. The trained model runs several orders of magnitude faster than a numerical simulator which allows to solve inverse problems requiring multiple calculation of forward solution. The global problem for the whole reservoir is reduced to an inverse problem. This problem is formulated as to find such unsteady boundary conditions p_{bc} for the single-well problems that will correspond to the given permeability field k, well bottomhole pressure p_{bhp} and production rate q and also will correspond to global pressure solution P formed by joining single-well solutions p. We optimize boundary condition p_{bc} via weighted-mean based local adaptation. The approach allows to obtain physically-realistic pressure fields but by now the error in production rates is quite big and equals to 50 %.

In the future work, we will extend the forward problem to multiphase flows and continue developing the methodology of the inverse problem solution to account better for production rates. Also we will consider extension to three-dimensional setup.

Acknowledgments

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