Data-Driven Model for Swirling Jet Atomization

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Abstract. This study reduces high-dimensional spatiotemporal data from Stereo-Particle Image Velocimetry (SPIV) to the dynamics of a few fundamental modes using Multi-resolution Dynamic Mode Decomposition (mrDMD). This approach aids in creating a reduced-order model and categorizing the various instabilities of ongoing liquid atomization. The pressure-swirl atomizer used in this study generated a well-defined air-core structure, and the SPIV measurements successfully captured the atomized jet's mean axial and radial velocity fields. The mrDMD analysis identified dominant flow structures by separating slow and fast spatiotemporal modes. The frequency-time analysis revealed three primary dynamic modes ranging from 0.2 Hz to 1.66 Hz, with a 0 Hz mode representing the steady-state flow. The extracted time series exhibited a "beating wave" pattern, indicating periodic perturbations in the atomization process. Additionally, the 0 Hz mode shape strongly correlated with the mean velocity contours captured by SPIV, reinforcing the capability of mrDMD to preserve essential dynamic features.

1. Introduction

Liquid fuels are atomized using various types of atomizers. These atomizers utilize high-pressure injection in pressure atomizers or the interaction of liquid fuel with gas flows in air-blast atomizers [1]. Although research on atomization has been ongoing for a long time, our physical understanding remains limited. The issues surrounding liquid breakup and droplet generation mechanisms near the nozzle remain unresolved in fluid mechanics [2]. This process breaks the continuous liquid flow from atomizers into fragments, which undergo secondary atomization to create a stable spray. The evaluation relies on empirical correlations specific to the geometry of each atomizer [2-4]. Pressure swirlers are extensively utilized across various industries, including chemical processes and combustion applications. Flow visualization techniques, such as Stereo-Particle Image Velocimetry, are employed to analyze the atomization process to measure droplets' velocity distribution [5,6]. Despite numerous investigations, there are still uncertainties about the accuracy of the scaling parameters that describe the atomization process. More analysis is necessary to understand the complex characteristics of swirling atomized flow. Cozzi et al. [7] utilized the Proper Orthogonal Decomposition (POD) technique on a swirl atomizer using SPIV, resulting in different flow modes. They detected strong oscillations in velocity that indicated the presence of a precessing core vortex (PVC) in the flow. The phase-averaged analysis of the velocity also identified a periodically varying, asymmetric wave pattern that the SPIV results alone could not capture. In contrast to traditional methods like Proper Orthogonal Decomposition (POD), recent advancements in data-driven modeling provide a deeper understanding of flow structures. Multiresolution Dynamic Mode Decomposition (mrDMD) was developed as a recursive extension of the standard Dynamic Mode Decomposition (DMD). This algorithm captures both slow and fast spatiotemporal modes. For example, mrDMD was used on sea surface data from 1990 to 2010 and determined the 1997 El Nino mode in Peru. In controlled experiments in the laboratory, mrDMD proved successful in identifying the modes of a liquid jet in crossflow. Rudy et al. [9] captured images of the liquid in a crossflow jet (LJIC) using a high-speed camera and analyzed them with mrDMD. The algorithm helped observe several new phenomena, such as ligament shedding, bag formations, and surface stripping. Developments in data-driven PDE discovery techniques [9] such as mrDMD can be used to characterize complex flows at greater depths. PDEFind is a partial differential equation algorithm that has successfully reconstructed the Navier-Stokes equation with an accuracy of 6%, the

diffusion equation with 2.4% accuracy, and Schrödinger's equation with 7% accuracy. The successful application of these techniques to experimental data on complex swirl atomization will enhance our understanding of these flows, which is essential for designing and optimizing atomizers.

2. Methodology

2.1. Experimental Setup

Experiments were conducted on a pressure-swirl atomizer. The atomizer has an orifice diameter of 1.35 mm and eight vanes, which generate the swirling atomized flow. DI water is pumped from a 4L tank using an AEM Electronics recirculation pump, with a maximum flow rate of 37 lpm and a maximum outflow pressure of 200 psi. The injected flow rate is measured using a Picomag magnetic flowmeter, which measures up to 35 lpm, and the injection pressure is measured using an Omega PX409 pressure transducer with a measurement range of up to 1000 psi. Figure 1 shows a schematic of the experimental setup. In this study, experiments were conducted at an injection pressure of 0.9 MPa and a flow rate of 11 ml/s. This pressure was selected based on its relevance to practical atomization processes and its ability to produce clear flow fields appropriate for the SPIV system.

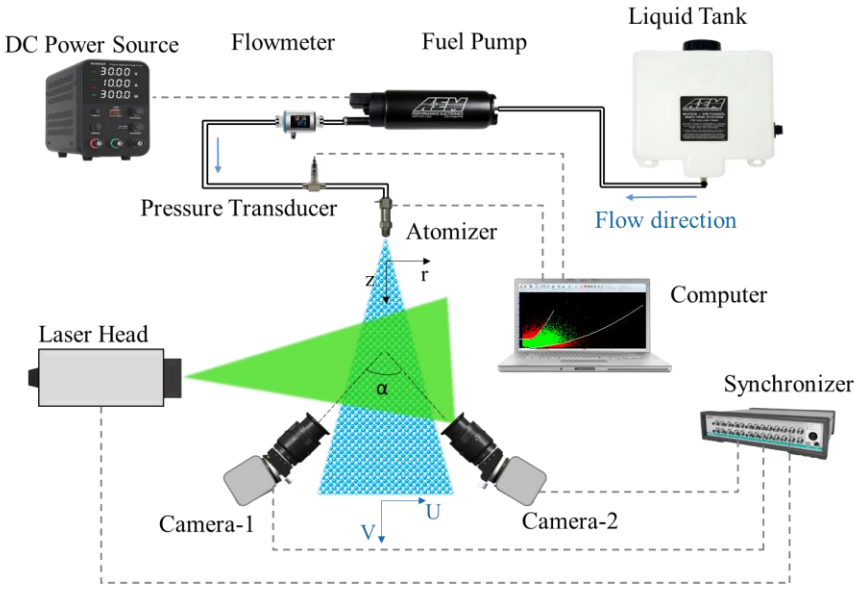


Figure 1 Experimental Setup Schematic

2.2. Stereo Particle Imaging Velocimetry

The swirling flow velocity contours were obtained using a Stereo Particle Image Velocimetry (SPIV) setup. A high-powered Nd:YAG pulsed laser (Dual Power) was used to generate two laser sheets, each 1.5 mm thick, with a wavelength of 532 nm and an energy output of 500 mJ/pulse. These laser sheets were aligned with the center of the nozzle to illuminate the flow. Two CCD cameras, positioned 30 cm apart, captured the flow field. The nozzle was placed vertically halfway between the cameras. Images were captured with a resolution of 2532 ×1768 pixels, representing a field of view of 68 × 50 mm^2. Dantec Dynamic Studio 7.6 controlled the SPIV setup and processed the captured images. The flow was captured at a rate of 5 Hz with a time pulse of 90 µs, and each camera recorded 800 images. The captured images were masked and had the background removed to eliminate noise. The *Adaptive PIV* function was used to generate the 2D velocity vector field, while *Universal Outlier Detection* and *Moving Average* were applied to reduce noise in the vector fields. Finally, the *Stereoscopic PIV* function was used to calculate the final instantaneous velocity contours. The Vector Statistics function was then applied to compute the final mean contours of axial (V), and radial (U) velocity components (Figure 2).

2.3. Multi-Resolution Dynamic Mode Decomposition

The Multi-Resolution Dynamic Mode Decomposition (mrDMD) method was developed by Kutz et al. [8], as a recursive application of Dynamic Mode Decomposition (DMD). This method segments the flow dynamics into different time scales and levels, separating slow "standing" structures from fast modes. Each mode is categorized into specific frequency ranges at different levels. mrDMD is an iterative process where, at each level, the time domain is split in half; specifically, each level has 2^{n-1} time windows where n is the level number. Then, DMD is applied to each sub-window at every level. This results in a hierarchical decomposition, capturing slow modes at progressively finer time resolutions. The linear expansion of mrDMD is given by:

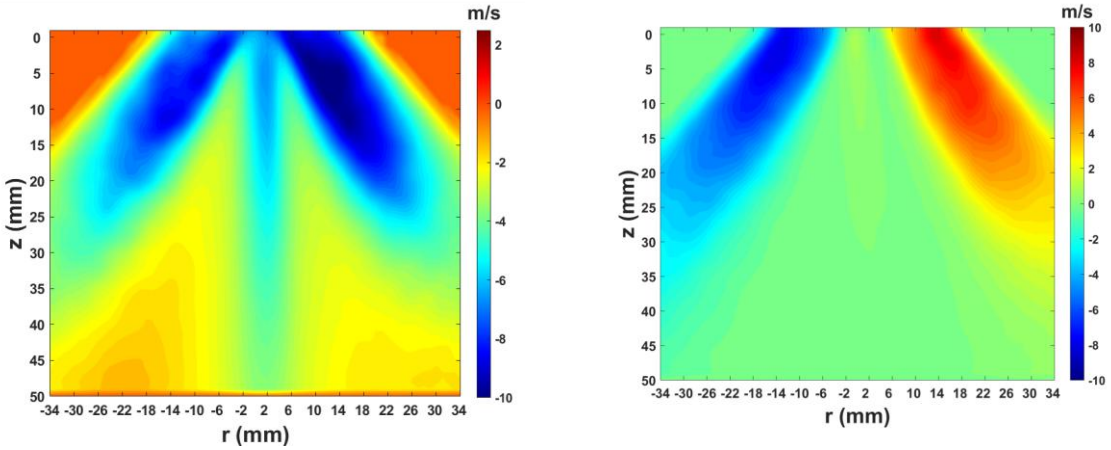
$$x(t) = \sum_{l=1}^{L} \sum_{j=1}^{J} \sum_{m=1}^{M} P^{l,j} \Phi_m^{l,j} e^{\omega^{l,j} t} \tag{1}$$

Where l represents the frequency levels, j denotes the time windows per level and m is the number of modes extracted at each level and window. To build a reduced model of the velocity contours, 9 levels and a rank of 20 were set. The analysis was performed on the instantaneous velocity contours obtained from the SPIV data. At each level, time windows are examined by comparing the window frequency with the cut-off frequency of that level; windows with frequencies higher than the cutoff frequency are considered to build the mode at that level. The time series of each mode is visualized to study its characteristics.

3. Results

3.1. SPIV Results

The velocity measurements were focused on the atomized region covering portions of the spray at 10 mm downstream from the nozzle. The mean velocity contours captured by the SPIV system are shown in Figure 2. As expected, axial and radial velocity contours show an axial symmetry around the nozzle's central vertical axis. The maximum axial velocity (V_{max} = 9 m/s) is measured at the top side of the velocity contour, close to the nozzle. The radial velocity contour (Figure 2b) highlights the outward expansion characteristic of a swirling atomized jet. A common feature observed across both contours is the distinct air-core interface outline, which clearly indicates that the nozzle effectively generates two jets in that plane, with an air core between them.



a) Axial Velocity (V) *b) Radial Velocity (U)*

Figure 2 Mean Velocity Contours

3.2. MrDMD Analysis

3.2.1. Frequency-Time Analysis

Figure 3 presents the frequency-time map, which illustrates the amplitude of each mode extracted from the mrDMD analysis. All modes fall within the 0.2 Hz to 1.66 Hz range, with the 0 Hz-level mode representing the standing "mean" mode. To properly capture these modes, the flow must be sampled at a frequency greater than 3.32 Hz, which is successfully done by the SPIV setup.

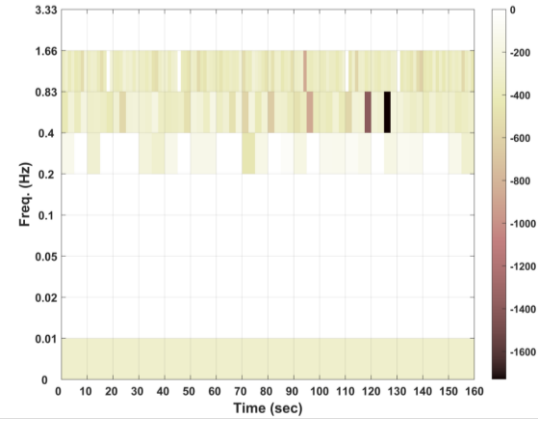


Figure 3 Frequency Time Map from mrDMD

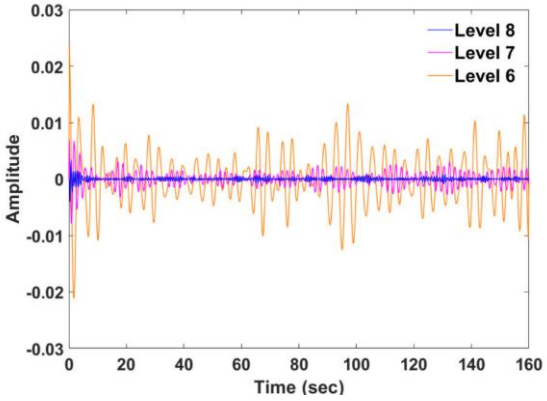


Figure 4 Time Series of Level 6,7,8

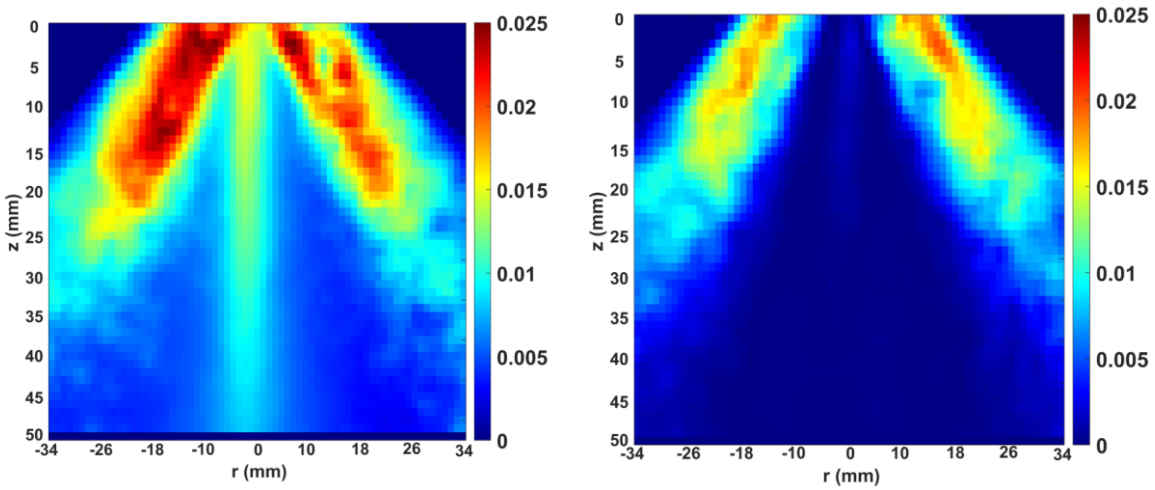
3.2.2. Reduced model

At each level, frequencies of time windows were compared with the level cutoff frequency to filter out weak modes and noise. To do that, the instantaneous velocity values from the SPIV were arranged in a matrix as follows:

$$\begin{bmatrix} u_n^i & \dots & u_n^T \\ \vdots & \dots & \vdots \\ u_N^i & \dots & u_N^T \\ v_n^i & \dots & v_n^T \\ \vdots & \dots & \vdots \\ v_N^i & \dots & v_N^T \end{bmatrix} \tag{2}$$

Where *n* represents a velocity at a specific (x,y) location in the velocity contour, *i* denotes the time step, *T* is the final time, and *N* is the total number of elements in the grid. Then, the above matrix was passed through the mrDMD function [8], and the time series of the three levels of interest were visualized and presented in **Error! Reference source not found.**. The first main feature is that all the modes exhibited a "beating wave" pattern, with the amplitudes decreasing with increasing levels. The frequency of the three modes is as follows: (i) level 6 (0.2-0.4 Hz), (ii) level 7 (0.4-0.83 Hz), and (iii) level 8 (0.83-1.66 Hz).

The time series of the 0 Hz mode was a flatline, which is expected from the 0 Hz mode; however, it is interesting to visualize the Phi shapes of the mode. Figure 5 shows the Phi shape of the 0 Hz mode, we can see that the mode has the same structure as both the axial and radial velocities visualized by the SPIV system.



a) Axial Velocity Phi Mode b) Radial Velocity Phi Mode

Figure 5 Phi shape of 0 Hz mode

4. Conclusion

This study investigated the swirling atomized flow generated by a pressure-swirl atomizer using Stereo Particle Image Velocimetry (SPIV) and Multi-Resolution Dynamic Mode Decomposition (mrDMD) for model reduction. The SPIV system successfully captured the velocity fields, providing detailed insights into the axial, radial, and tangential velocity components. The frequency-time analysis revealed three dominant modes along with the 0 Hz mode. The time series of the modes was used to investigate the behavior of the underlying structures, which showed a beating phenomenon. The 0 Hz mode showed to have the same structure as the visualized flow from the SPIV. The results confirm that mrDMD provides an effective framework for reducing complex swirling flows while preserving key dynamic features. For future work, the reduced models could be used to develop data-driven partial differential equations for numerical representations of the flow.

5. References

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